Important equivalences between CTL formulas:

Two CTL formulas φ and ψ are said to be semantically equivalent if any state in any model which satisfies one of them also satisfies the other; we denote this by $\varphi \equiv \psi$.

We have already noticed that A is a universal quantifier on paths and E

is the corresponding existential quantifier. Moreover, G and F are also universal and existential quantifiers, ranging over the states along a particular

path. In view of these facts, it is not surprising to find that de Morgan rules

exist:

$\neg AF\ \varphi \equiv EG\ \neg\varphi$

$\neg EF\ \varphi \equiv AG\ \neg\varphi$ (3.6)

$\neg AX\ \varphi \equiv EX\ \neg\varphi$.

We also have the equivalences

$AF\ \varphi \equiv A[\ U\ \varphi]$ $EF\ \varphi \equiv E[\ U\ \varphi]$

which are similar to the corresponding equivalences in LTL

Adequate sets of CTL connectives

As in propositional logic and in LTL, there is some redundancy among the CTL connectives. For example, the connective AX can be written $\neg EX\ \neg$; and AG, AF, EG and EF can be written in terms of AU and EU as follows:

first, write AG φ as $\neg EF\neg\varphi$ and EG φ as $\neg AF\ \neg\varphi$, using, and then use $AF\ \varphi \equiv A[\ U\ \varphi]$ and $EF\ \varphi \equiv E[\ U\ \varphi]$. Therefore AU, EU and EX form an adequate set of temporal connectives.

Also EG, EU, and EX form an adequate set, for we have the equivalence

$A[\varphi\ U\ \psi] \equiv \neg(E[\neg\psi\ U\ (\neg\varphi \wedge \neg\psi)] \vee EG\ \neg\psi)$ which can be proved as follows:

$A[\varphi\ U\ \psi] \equiv A[\neg(\neg\psi\ U\ (\neg\varphi \wedge \neg\psi)) \wedge F\ \psi]$

$\equiv \neg E\neg[\neg(\neg\psi\ U\ (\neg\varphi \wedge \neg\psi)) \wedge F\ \psi]$

$\equiv \neg E[(\neg\psi\ U\ (\neg\varphi \wedge \neg\psi)) \vee G\ \neg\psi]$

$\equiv \neg(E[\neg\psi\ U\ (\neg\varphi \wedge \neg\psi)] \vee EG\ \neg\psi)$.

The first line is by Theorem 3.10, and the remainder by elementary manipulation. (This proof involves intermediate formulas which violate the syntactic formation rules of CTL.